POROUS FLOW AT A VERTICAL SURFACE IN FREE CONVECTION (LAMINAR BOUNDARY LAYER).

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An approximate method is developed for calculating heat transfer at a vertical surface in free convection with uniform inward and outward porous flow and a laminar boundary layer. Design formulas are given, and the results obtained are analyzed.

Various means of cooling are now being used to protect structural elements from overheating, the most common being transpiration cooling by injecting gas into the boundary layer. Whereas numerous methods of calculating this kind of cooling have been developed for forced convection, the same cannot be said of natural convection.

Let us examine a vertical porous surface at a constant temperature T_W. Suppose a plate is located in a stationary fluid, whose temperature at a large distance from the wall is T_∞. Let $T_W > T_\infty$ (the argument is unchanged if $T_\infty > T_W$).

Fluid is supplied through the porous plate at a velocity v_W constant over the whole surface. The added fluid has a temperature equal to that of the wall T_W and the same physical properties as the principal medium.

The boundary layer equation may then be written as:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2}, \tag{1}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2}, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{3}$$

Let us assume that all the physical properties of the fluid are independent of temperature, and that the dissipation term in natural convection is negligibly small.

The boundary conditions are:

$$\begin{aligned} &\text{when } y = 0 \quad u = 0, \quad v = v_w, \quad T = T_w; \\ &\text{when } y = \infty \quad u = 0, \quad T = T_\infty. \end{aligned} \tag{4}$$

Integrating (1)-(3) over the boundary layer thickness $\delta(x)$, and taking into account boundary conditions (4), we obtain the integral relations

$$\frac{d}{dx}\int_0^\delta u^2 dy = g\beta\int_0^\delta \theta\, dy - \nu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{5}$$

$$\frac{d}{dx}\int_0^\delta \theta\, u\, dy = -a\left(\frac{\partial\theta}{\partial y}\right)_{y=0} + v_w\theta_w. \tag{6}$$

In (5) and (6), $\theta = T - T_\infty$ and $\theta_W = T_w - T_\infty$ for the case $T_W > T_\infty$; $\theta = T_\infty - T$ and $\theta_W = T_\infty - T_w$ when $T_W < T_\infty$. For injection v_W has a positive sign, and for suction a negative one.

We shall use the Karman-Polhausen method to solve (5) and (6), and take the velocity and temperature profiles in the boundary layer in the form of fourth-degree polynomials:

$$u = a_0 + a_1\left(\frac{y}{\delta}\right) + a_2\left(\frac{y}{\delta}\right)^2 + a_3\left(\frac{y}{\delta}\right)^3 + a_4\left(\frac{y}{\delta}\right)^4, \tag{7}$$

$$T = b_0 + b_1\left(\frac{y}{\delta}\right) + b_2\left(\frac{y}{\delta}\right)^2 + b_3\left(\frac{y}{\delta}\right)^3 + b_4\left(\frac{y}{\delta}\right)^4. \tag{8}$$

To find the coefficients in (7) and (8), we use the boundary conditions:

$$\begin{aligned} &\text{when } y = 0 \quad u = 0, \; v_w\left(\frac{\partial u}{\partial y}\right)_{y=0} = \nu\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} + g\beta(T_w - T_\infty), \\ &T = T_w, \; v_w\left(\frac{\partial T}{\partial y}\right)_{y=0} = a\left(\frac{\partial^2 T}{\partial y^2}\right)_{y=0}; \\ &\text{when } y = \delta \quad u = 0, \; \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} = 0, \\ &T = T_\infty, \; \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} = 0. \end{aligned} \tag{9}$$

Using (9), we obtain the following expressions for the velocity and temperature distributions in the boundary layer:

$$u = \frac{g\beta\theta_w\delta^2}{(6\nu + v_w\delta)}\left(\frac{y}{\delta}\right)\left(1 - \frac{y}{\delta}\right)^3, \tag{10}$$

$$\theta = \theta_w\left[1 - \frac{12a}{(6a + v_w\delta)}\left(\frac{y}{\delta}\right) - \frac{6v_w\delta}{(6a + v_w\delta)}\left(\frac{y}{\delta}\right)^2 + \frac{12a + 8v_w\delta}{(6a + v_w\delta)}\left(\frac{y}{\delta}\right)^3 - \frac{6a + 3v_w\delta}{(6a + v_w\delta)}\left(\frac{y}{\delta}\right)^4\right]. \tag{11}$$

Replacing u and θ in (6) by the corresponding expressions determined from (10) and (11), and integrating over the limits 0 to δ, we obtain a differential equation relating the boundary layer thickness to the coordinate x and the injection (or suction) velocity:

$$\frac{d}{dx}\left\{\frac{g\beta\theta_w^2\delta^3(66a + 15v_w\delta)}{504(6\nu + v_w\delta)(6a + v_w\delta)}\right\} = \theta_w\frac{(12a^2 + 6av_w\delta + v_w^2\delta^2)}{\delta(6a + v_w\delta)}. \tag{12}$$

The solution of (12) has the form

$$\begin{aligned} \left(\frac{g\beta\theta_w}{504}\right)\Bigg\{ & 15\frac{\delta^2}{v_w^2} - \frac{\nu}{v_w^3}(24a + 90\nu)\delta + \frac{A}{v_w}\ln\left(6 + \frac{v_w\delta}{\nu}\mathrm{Pr}\right) + \\ & + \frac{B_1}{v_w}\ln\left(6 + \frac{v_w\delta}{\nu}\right) - \frac{B_2}{v_w\nu}\left(6 + \frac{v_w\delta}{\nu}\right)^{-1} + \\ & + \frac{C}{v_w^2}\left[\frac{1}{2}\ln\left(12 + 6\frac{v_w\delta}{\nu}\mathrm{Pr} + \frac{v_w^2\delta^2}{\nu^2}\mathrm{Pr}^2\right) - \right. \\ & \left. - \frac{3}{\sqrt{3}}\operatorname{arctg}\left(\frac{v_w\delta}{\nu}\mathrm{Pr} + 3\right)3^{-\frac{1}{2}}\right] + \\ & + \frac{D\,\mathrm{Pr}}{v_w\nu}\frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{v_w\delta}{\nu}\mathrm{Pr} + 3\right)3^{-\frac{1}{2}}\Bigg\} - \mathrm{const} = x, \end{aligned} \tag{13}$$

where

$$\begin{aligned} \mathrm{const} = \left(\frac{g\beta\theta_w}{504}\right)\Bigg\{ & \frac{1.792(A + B_1)}{v_w} - \frac{0.166B_2}{v_w\nu} - \\ & - \frac{0.5716C}{v_w^2} + \frac{0.6048D\,\mathrm{Pr}}{v_w\nu}\Bigg\}. \end{aligned} \tag{14}$$

Expression (13) contains an implicit expression for the dependence of the boundary layer thickness $\delta(x)$ on the injection (or suction) velocity.

The coefficients $A,\ B_1,\ B_2,\ C$ and D are solutions of the following inhomogeneous system of linear algebraic equations:

$$A + B_1 + \frac{1}{v_w} C = -360\, a^2 \frac{1}{v_w^3};$$

$$(6a + 12\nu) A + (12a + 6\nu) B_1 + B_2 + D + \frac{(12\nu + 6a)}{v_w} C =$$

$$= \frac{1}{v_w^3} (-1008a^3 - 2376a^2\nu + 864a\nu^2 + 3240\nu^3);$$

$$(12a^2 + 72a\nu + 36\nu^2) A + (48a^2 + 72a\nu) B_1 + 12aB_2 + (12\nu + 6a) D +$$

$$+ \frac{(72a\nu + 36\nu^2)}{v_w} C = \frac{1}{v_w^3} (-5616a^3\nu + 10368a^2\nu^2 + 38880a\nu^3 + 1728a^4); \qquad (15)$$

(15)

$$(144a^2\nu + 216a\nu^2) A + (72a^3 + 288a^2\nu) B_1 + 48a^2 B_2 + (72a\nu + 36\nu^2) D +$$

$$+ \frac{216a\nu^2}{v_w} C = \frac{1}{v_w^3} (20736a^4\nu + 41472a^3\nu^2 + 155520a^2\nu^3);$$

$$432a^2\nu^2 A + 432a^3\nu B_1 + 72a^3 B_2 + 216a\nu^2 D = \frac{2592a^3\nu^2(24a + 90\nu)}{v_w^3}.$$

In order to obtain from (13), bearing in mind (14) and (15), an explicit expression for the boundary layer thickness as a function of the injection (or suction) velocity and the coordinate x, let us write the left hand side of (13) in the form of a Maclaurin series.

Denoting the left hand side of (13) by $F(\delta,\ v_w,\ \mathrm{Pr})$ for a fixed injection (or suction) velocity v_W and Pr number we have:

$$F(\delta,\ v_w,\ \mathrm{Pr}) = \sum_{n=0}^{n=\infty} \frac{\delta^n}{n!} f^{(n)}(\delta = 0,\ v_w,\ \mathrm{Pr}), \qquad (16)$$

where $f^{(n)}(\delta = 0,\ v_w,\ \mathrm{Pr})$ is the n-th derivative of the function $F(\delta,\ v_w,\ \mathrm{Pr})$ with respect to δ.

The first four terms of (16) vanish while the fifth and subsequent terms are nonzero. For $v_W > 0$, the even terms of the series, beginning with $f^{(4)}(\delta = 0,\ v_w,\ \mathrm{Pr})$, are positive, while the odd terms, beginning with $f^{(5)}(\delta = 0,\ v_w,\ \mathrm{Pr})$ are negative. In the suction case ($v_W < 0$) all the terms of **(16)** are positive.

Through transformation of the power series, the boundary layer thickness may be obtained in explicit form as a function of the coordinate x for a given injection (or suction) velocity.

For large injection (suction) velocities (16) diverges, and it is therefore necessary to determine the radius of convergence as a function of the injection (suction) velocity for each Pr number.

Let us denote the maximum injection (suction) velocity for which (16) converges by $v_{W\ max}$. Then, for Pr = 0.72 and Pr = 1.0, the series converges if the quantity $|v_{W\ max}\ \delta/\nu|$ entering into the remaining term, is less than 3.46 or 2.15, respectively.

By estimating the remaining term in expansion (16) it is possible to limit the calculations to the first two nonzero terms. The error in boundary layer thickness evaluated from only two terms of (16) in this way does not exceed 10%, if the injection (suction) parameter determined by (21') lies within the following limits:

$$\eta = \left|\frac{v_w x}{\nu}\right| \left(\frac{\mathrm{Gr}_x}{4}\right)^{-1/4} \leqslant 0.75 \quad \text{for} \quad \mathrm{Pr} = 0.72,$$

$$\eta = \left|\frac{v_w x}{\nu}\right| \left(\frac{\mathrm{Gr}_x}{4}\right)^{-1/4} \leqslant 0.70 \quad \text{for} \quad \mathrm{Pr} = 1.0.$$

Bearing these remarks in mind, we have

$$\delta = \left(\frac{g\beta\theta_w}{\nu^2}\right)^{-1/4} x^{1/4} \left(\frac{504\cdot 4!}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right)^{1/4} \times$$

$$\times \left(1 - \frac{\mathrm{Re}_\delta}{5} \frac{f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr})}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right)^{-1/4}. \tag{17}$$

Since the term $\frac{\mathrm{Re}_\delta}{5} \frac{f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr})}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}$ is less than unity, we obtain

$$\left(\frac{\delta}{x}\right) = \mathrm{Gr}_x^{-1/4} \left(\frac{504\cdot 4!}{f^{(4)}\,\delta=0,\ v_w,\ \mathrm{Pr})}\right)^{1/4} \left(1 + \frac{\mathrm{Re}_\delta\, f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr})}{20\, f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right) \tag{18}$$

or

$$\left(\frac{\delta}{x}\right) = \mathrm{Gr}_x^{-1/4} \left(\frac{504\cdot 4!}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right)^{1/4} \times$$

$$\times \left(1 - \frac{f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr})\,\mathrm{Re}_x(\mathrm{Gr}_x/4)^{-1/4}}{20\sqrt[4]{4}\, f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right)^{-1}. \tag{19}$$

In (17), (18) and (19) $\mathrm{Gr}_x = g\beta\theta_w x^3/\nu^2$, $\mathrm{Re}_\delta = v_w\delta/\nu$, $\mathrm{Re}_x = v_w x/\nu$ and

$$f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr}) = \left(\frac{g\beta\theta_w}{504}\right)\left(\frac{v_w}{\nu}\right)^4 \left\{-\frac{A\,\mathrm{Pr}^4\, 3!}{v_w\, 6^4} - \frac{B_1\, 3!}{v_w\, 6^4} - \right.$$

$$\left. - \frac{B_2\, 4!}{v_w\, \nu\, 6^5} + \frac{\mathrm{Pr}^2\,[2304C\,\mathrm{Pr}^2 + 576C\,\mathrm{Pr}^3 - 432\,(v_w/\nu)\, D\,\mathrm{Pr}^3]}{v_w^2\,(12 + 6\mathrm{Re}_\delta\,\mathrm{Pr} + \mathrm{Re}_\delta^2\,\mathrm{Pr}^2)^4}\right\}, \tag{20}$$

$$f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr}) = \left(\frac{g\beta\theta_w}{504}\right)\left(\frac{v_w}{\nu}\right)^5 \left\{\frac{A\,\mathrm{Pr}^5\, 4!}{v_w\, 6^5} + \frac{B_1\, 4!}{v_w\, 6^5} + \frac{B_2\, 5!}{v_w\, \nu\, 6^6} + \right.$$

$$\left. + \frac{\mathrm{Pr}^2\,[-41472C\,\mathrm{Pr}^3 - 12096C\,\mathrm{Pr}^4 + 3456\,(v_w/\nu)\, D\,\mathrm{Pr}^4]}{v_w^2\,(12 + 6\mathrm{Re}_\delta\,\mathrm{Pr} + \mathrm{Re}_\delta^2\,\mathrm{Pr}^2)^5}\right\}. \tag{21}$$

We shall denote the product $\mathrm{Re}_x(\mathrm{Gr}_x/4)^{-1/4}$ by η and call it the injection (suction) parameter,

$$\eta = \mathrm{Re}_x\,(\mathrm{Gr}_x/4)^{-1/4}. \tag{21'}$$

The quantity η in (21') coincides with the variable ζ of [3].

The local heat transfer coefficient is determined from the relation $\alpha = -\frac{\lambda}{(T_w - T_\infty)} \operatorname{grad}\theta\Big|_w$ or in dimensionless form:

$$\mathrm{Nu}_x = -\frac{x}{(T_w - T_\infty)} \operatorname{grad}\theta\Big|_w. \tag{22}$$

Substituting in (22) the value of the temperature gradient at the wall from (11), we have

$$\mathrm{Nu}_x = \left(\frac{x}{\delta}\right) \frac{12}{(6 + \mathrm{Re}_\delta\,\mathrm{Pr})}. \tag{23}$$

Substituting in the last equation the boundary layer thickness δ from (19), we obtain

$$\mathrm{Nu}_x = 12\mathrm{Gr}_x^{1/4} \left[\frac{504\cdot 4!}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right]^{-1/4} \left[1 - \frac{\eta\, f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr})}{20\sqrt[4]{4}\, f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right] \times$$

$$\times \left\{6 + \left[\left(\frac{504\cdot 4!}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right)^{1/4} \mathrm{Pr} - \frac{3f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr})}{10f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right] \frac{\eta}{\sqrt[4]{4}}\right\}^{-1}. \tag{24}$$

When $v_W = 0$, (24) becomes

$$\mathrm{Nu}_{x_0} = 2\mathrm{Gr}_x^{1/4} \left[\frac{504\cdot 4!}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right]^{-1/4} \tag{25}$$

which coincides with Polhausen's exact solution [7] for a laminar boundary layer and natural convection at an impermeable surface.

Let us use (25) as a standard in determining the influence of injection (suction) on heat transfer in natural convection.

The dependence of Nu_x/Nu_{x_0} on the injection (suction) parameter $\eta = Re_x (Gr_x/4)^{-1/4}$. is given in Fig. 1.

The authors of [3] obtained a computer solution in the form of an infinite series, as in [2], and confined themselves to the first term of the expansion, which leads to discrepancies of 10%, compared to our results, for the maximum suction velocity (η = – 0.8) and 4.5% for injection (η = 0.8).

Our results show good agreement with those of [2] and [3].

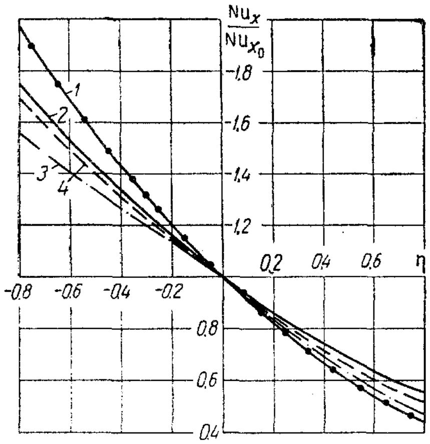

Fig. 1. Influence of injection (and suction) on local heat transfer in natural convection: v_W = const, 1 – according to (24) with Pr = 1.0, and 2 – with Pr = 0.72; 3 – according to [3], with Pr = 0.72, $v_W \sim x^{-1/4}$, T_W = const; 4 – according to [2], with Pr = 0.72.

It can be seen from Fig. 1 that the influence of injection (suction) on heat transfer increases with the Pr number.

Transforming (19) into

$$\left(\frac{y}{\delta}\right) = \left(\frac{y}{x}\right)\left(1 - \frac{f^{(5)}(\delta = 0,\ v_w,\ \mathrm{Pr})}{20\sqrt[4]{4}\, f^{(4)}(\delta = 0,\ v_w,\ \mathrm{Pr})}\,\eta\right) \times \mathrm{Gr}_x^{1/4}\left(\frac{504\cdot 4!}{f^{(4)}(\delta = 0,\ v_w,\ \mathrm{Pr})}\right)^{-1/4} \qquad (26)$$

and substituting the latter in (11), we obtain the dimensionless temperature distribution in the boundary layer for injection and suction. This dependence is shown in Fig. 2a as a function of the coordinate $\zeta = \left(\frac{y}{x}\right)\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}$ for various values of the injection (suction) parameter and Pr = 0.72. The data of [3] are also included in Fig. 2a for comparison purposes.

The influence of the Pr number on the temperature profile is shown in Fig. 2b in the same coordinates, together with Polhausen's exact solution [7] for v_W = 0.

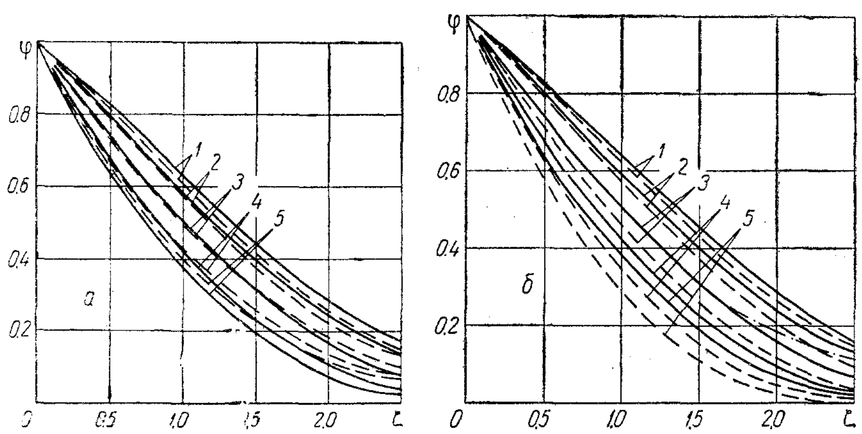

Fig. 2. Temperature distribution in boundary layer $\varphi = \theta/\theta_w$: a – according to (11) solid line for Pr = 0.72, dotted line according to [3]; b – solid line for Pr = 0.72, dotted line for Pr = 1.0 according to (11), and dot-dash line according to [7] for η = 0. 1 – η = 0.6; 2 – 0.4; 3 – 0; 4 – (– 0.4); 5 – (– 0.6)

The dimensionless temperature profiles for injection parameters $\eta \approx 0.6$ and above have a point of inflection, which is in complete qualitative agreement with the analogous case for forced convection.

The velocity distribution in the boundary layer may be obtained from (26) and (19) after substitution in (10), and in dimensionless form this may be written:

$$\frac{ux/\nu}{\mathrm{Gr}_x^{1/2}} = \zeta\left[\frac{504\cdot 4!}{f^{(4)}(\delta = 0,\ v_w,\ \mathrm{Pr})}\right]^{1/4} \times$$

$$\times\left\{1-\zeta\left(1-\frac{f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr})}{20\sqrt[4]{4}f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\eta\right)\left[\frac{504\cdot 4!}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right]^{-1/4}\right\}^3\times$$

$$\times\left\{6+\frac{\eta\,\mathrm{Pr}}{\sqrt[4]{4}}\left[\frac{504\cdot 4!}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right]^{1/4}-\frac{3\eta}{10\sqrt[4]{4}}\,\frac{f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr})}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right\}^{-1}. \quad (27)$$

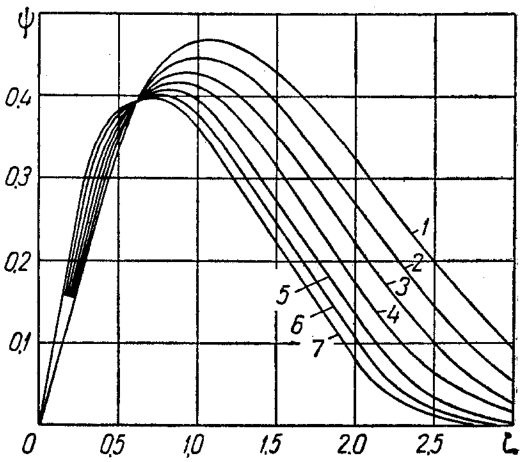

Fig. 3. Velocity distribution in boundary layer according to (27) for Pr = 1.0: 1 – η = 0.6; 2 – 0.4; 3 – 0.2; 4 – 0; 5 – (–0.2); 6 – (–0.4); 7 – (–0.6)

Fig. 3 shows the dimensionless velocity $\psi=\left(\frac{ux}{\nu}\right)\mathrm{Gr}_x^{-1/2}$ at various values of the injection (suction) parameter η for Pr = 1.0. It can be seen that with increased injection the velocity profile becomes fuller, and the maximum velocity and its distance from the wall increase. For suction the picture is reversed – the velocity profile becomes less full, and its maximum falls and moves closer to the wall. This behavior of the velocity profile affects the tangential stresses.

Knowing the velocity distribution in the boundary layer, we can calculate the shear stress at the wall from the equation:

$$\tau=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}. \quad (28)$$

If we put the injection (suction) velocity equal to zero in all the above calculations, we obtain the results for free convection at an impermeable wall. For example, according to (17) and (21) the boundary layer thickness and the heat transfer for Pr = 0.72 will be

$$(\delta/x)=5.42\,\mathrm{Gr}_x^{-1/4}\,[1-0.268\,\mathrm{Re}_x\,(\mathrm{Gr}_x/4)^{-1/4}]^{-1}, \quad (29)$$

$$\mathrm{Nu}_x=0.369\mathrm{Gr}_x^{1/4}\,[1-0.268\mathrm{Re}_x(\mathrm{Gr}_x/4)^{-1/4}]^2\,[1+0.192\mathrm{Re}_x\,(\mathrm{Gr}_x/4)^{-1/4}]^{-1}. \quad (30)$$

When $v_w=0$ $(\delta/x)_0=5.42\,\mathrm{Gr}_x^{-1/4}$ and $\mathrm{Nu}_{x_0}=0.369\,\mathrm{Gr}_x^{1/4}$, which is 2.7% different from the exact solution [7].

Finally, we shall consider how the temperature and velocity profiles, substituted in (5) and (6), affect the final result for local heat transfer.

For this purpose we take the temperature and velocity distributions in the form of third-degree polynomials satisfying the boundary conditions (9), instead of (7) and (8). We then have:

$$u=\frac{g\beta\theta_w\delta^2}{(4\nu+v_w\delta)}\left(\frac{y}{\delta}\right)\left(1-\frac{y}{\delta}\right)^2, \quad (31)$$

$$\theta=\theta_w\left[1-\frac{6a}{(4a+v_w\delta)}\left(\frac{y}{\delta}\right)-\frac{3v_w\delta}{(4a+v_w\delta)}\left(\frac{y}{\delta}\right)^2+\right.$$

$$\left.+\frac{2(a+v_w\delta)}{(4a+v_w\delta)}\left(\frac{y}{\delta}\right)^3\right]. \quad (32)$$

Substitute (31) and (32) in (5) and (6). Then (19) and (20) take the form:

$$\left(\frac{\delta}{x}\right)=\left[\frac{210\cdot 4!}{f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right]^{1/4}\times$$

$$\times\,\mathrm{Gr}_x^{-1/4}\left[1-\frac{f^{(5)}(\delta=0,\ v_w,\ \mathrm{Pr})\,\mathrm{Re}_x\,(\mathrm{Gr}_x/4)^{-1/4}}{20\sqrt[4]{4}f^{(4)}(\delta=0,\ v_w,\ \mathrm{Pr})}\right]^{-1}, \quad (33)$$

$$\mathrm{Nu}_x=\left(\frac{x}{\delta}\right)\frac{6}{(4+\mathrm{Re}_\delta\,\mathrm{Pr})}. \quad (34)$$

For Pr = 0.72 we shall obtain

$$\mathrm{Nu}_x=0.36\,\mathrm{Gr}_x^{1/4}\,[1-0.353\mathrm{Re}_x(\mathrm{Gr}_x/4)^{-1/4}]^2\,[1+0.178\mathrm{Re}_x(\mathrm{Gr}_x/4)^{-1/4}]^{-1}. \quad (35)$$

The values of $f^{(5)}(\delta = 0,\ v_w,\ \mathrm{Pr})$ and $f^{(4)}(\delta = 0,\ v_w,\ \mathrm{Pr})$ in (33) are different from in (20) and (21). Comparison of the local heat transfer with [3] shows that in this case the discrepancy ≈ 17% for maximum suction velocities ($\eta = -0.8$), and ≈ 3% for maximum injection velocities.

NOTATION

v_w – injection or suction velocity; T – temperature; η – injection (suction) parameter; ζ – dimensionless coordinate; ψ – dimensionless boundary layer velocity; Re_x –Reynolds number based on injection (suction) velocity and coordinate x; Re_δ – Reynolds number based on injection (suction) velocity and boundary layer thickness.

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